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Collaborative Problem-Solving for the ‘Polycrisis’

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Abstract

The paper focuses on the “Polycrisis,” which is first defined and then positioned as the context for a subsequent discussion about both the formal and social conditions for effective collaborative problem-solving. It then highlights the potential for diagrammatic reasoning to contribute to solving what some commentators choose to call ‘wicked-problems’ including those associated natural, economic, and social forms of fragility. This discussion is informed by pertinent advances in applied category theory, which has a wide range of application-domains including cyber-physical systems, scientific systems, dynamical systems, software engineering, and machine learning. The formal constructs and techniques to be surveyed include optics and parametric lenses, as well as David Spivak’s organisational categories. Optics and lenses have not only been applied to software engineering but also to the modelling of dynamical systems and machine learning. They possess a diagrammatic representation (as string diagrams), which serves as an aid in their deployment (e.g. AlgebraicJulia, Symbolica AI, Haskell applications). The paper argues that these developments should assist users in their collaborative modelling of the *Polycrisis* by: (i) integrating machine learning, differential programming, and calibration and simulation of dynamical systems; (ii) accounting for subsystem interactions within a larger system that features different stock-flow rates; (iii) accommodating non-linear mappings and weight-sharing; (iv) formally supporting a variety of stochastic influences over the systems that are being modelled; and, (iv) imposing on the whole, the internal logic associated with a topos.

Keywords: Machine-learning, Polycrisis, Applied Category Theory, Diagrammatic Reasoning, Optics, Organizational Category,

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1. Introduction

The political and economic context for the paper is informed by the notion that "cracks" are finally occurring to the edifice erected after half a Century of Neoliberal policy. In a nutshell, politicians of all persuasions in the US, Europe and Australia are waking up to the fact that too much productive capacity has been "offshored", especially in the manufacturing sector. This loss of capacity has undermined the ability of national governments to innovate, including in the advanced manufacturing and defence equipment sectors, and the application of AI alone will not help much to overcome this weakness.

This problem is generally discussed in the context of "strategic supply chain management", but is something that can readily be investigate through comparative studies of R&D and other innovative activity across nations. Policy responses have included Biden's omnibus "Anti-Inflation Bill", Trump's MAGA threats to impose punitive tariffs on China, and the "Future Made in Australia" proposals of Albanese. For its part, China has attempted to transcend the dichotomy between "market versus plan" (Boer, 2024) and has embraced a technology-based industry policy under the influence of Germany's "Industrie-4.0" agenda (Naughten, 2021).

Nevertheless, the main focus of the paper will be on recent developments in machine-learning and dynamical systems modelling that have taken-off under the influence of applied category theory. To this end, pertinent mathematical and computational advances will be reviewed, which have emerged within a number of research institutes and advanced technology companies including Symbolic AI, the AlgebraicJulia group, the Topos Institute, and Quantinuum.

In this account, the powers of diagrammatic reasoning will feature strongly as we consider the formal analysis of processes of composition between sub-systems, and the role of particular constructs such as parametric lenses, Mealy and Moore machines, polynomial functors, bicategories, and the Org category. A schematic overview of pertinent themes and their interlinkages is presented in Figure. 1 below.

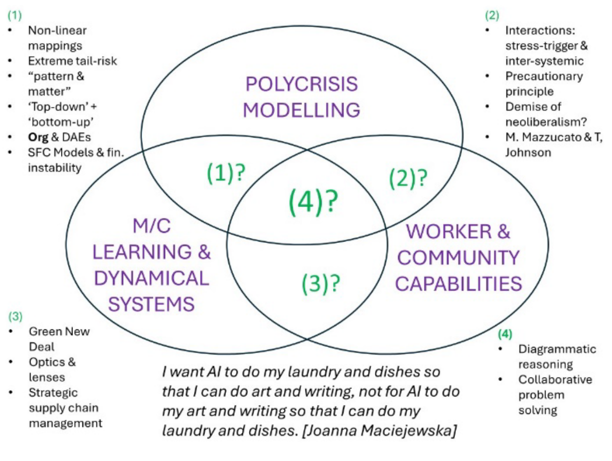


Figure 1- Themes and Linkages Relating to the Polycrisis

The focus of this paper will be on Collaborative Problem-Solving for the 'Polycrisis'. This conceptual framework is defined in section 2, and will provide the context for what is to follow. The role of applied category theory in accounting for compositionality and bijective flows of information both in machine learning and in dynamical system modelling will be introduced in section 3 and discussed in more detail in three subsections, Section 4 brings this discussion to a close by considering ways that diagrammatic reasoning can support collaborative forms of problem solving. Conclusions will follow in the final section 5 of the paper.

2. Polycrises as a complex system

Polycrises are characterised by an "array of grave, long-term challenges, including climate change, biodiversity loss, pandemics, widening economic inequalities, financial system instability, ideological extremism, and an escalating danger of nuclear war" (Cascade Institute, 2024).

Initially, it may help to define exactly what is meant by a complex system. David Rickles (2008) has proposed a set of inter-related characteristics: (i) a unit system must contain many subunits; (ii) These subunits must be interdependent at least some of the time; (iii) the interaction between subunits must be non-linear at least some of the time; (iv) properties of the system must be supervenient on those of the subunits and on their interactions. Moreover, these properties may be described as 'emergent,' when they amount to a new complex structure transcending those of the smaller subunits. Heterogeneity of subunits can contribute to this emergence. Typically, the subunits would modify their characteristics and behaviour both in response to changes in their environment (i.e. they are self-organizing adaptive) but also to changes in the system as a whole, which they might influence. Within economic and financial systems, other influences and drivers include network effects and the nature of non-linear interactions plus processes of herding and alignment, as well as speculative bubbles that may arise due to self-organizing behaviour (Rickles, 2008).

A crucial feature of crisis phenomena is extreme tail-risk and the modelling of financial, 'natural', and economic fragility. This includes the much-debated application of the precautionary principle to the characterisation of ecological sustainability.

The 2024 Cascade Institute Report, *Introduction to Polycrisis Analysis*, describes how relationships between stresses trigger events and crises across two or more interacting systems; suggesting that these can combine to create four broad causal pathways that "provide a 'grammar' for mapping the distinct system interactions that can form a polycrisis" (Lawrence et al., 2024).

i. Interacting stresses (stresses in one system amplify stresses in a second system, or the two systems have common stresses)

ii. Inter-systemic stress-trigger interactions (stresses in one system affect the trigger event of another system)

iii. Crisis impacts on adjacent systems (a crisis in one system may affect the stresses and/or trigger event of another system)

iv. Inter-systemic crisis interactions (a crisis in one system may interact with a crisis in another)

However, the formal tools promoted by the Cascade Institute could be construed as inadequate to the task of providing solutions to the Polycrisis. This helps to explain the paper's focus on current developments in the field of applied category theory.

3. New Mathematics Means New Forms of Modelling

The first section of the paper focusing on the polycrisis could be summarized by the maxim "Complex problems require complex but *integrated* solutions". The Introduction above emphasized the "cracks" appearing within Neoliberalism globally, because so much productive capacity in Australia, the US, and elsewhere had been 'offshored' and AI 'won't help much' in compensating for this loss.

There is now a dawning recognition that problems associated with "strategic supply-chain management" and the 'Green Transition' require what Mariana Mazzucato (Mazzucato et al., 2020) describes as the "Challenge-oriented Financing of Innovation." Meanwhile, Roland Boer (2024) explains how China managed to move beyond the dichotomous choice between "market versus plan" to plan and finance infrastructure developments and innovation, on both a national and global level (through BRICS lending and Belt-and-Road' initiatives).

Moreover, both the Chairman of the US Securities and Exchange Commission and Senior Managers from the European Central Bank and the IMF have warned that new AI algorithmics and 'fintech' developments could both *trigger-off* and *amplify* financial downturns while *prolonging* recovery times (Gensler and Bayley, 2020; Gopinath, 2024), while sounding warnings about the rapid expansion of unregulated private 'risk' in the Shadow Banking sector (Cohen et al., 8 April 2024).

From a sustainability perspective that is both environmental and economic, 'regenerative and sustainable agriculture' is under *severe pressure* from diseconomies of scale, fragmentation, and profit-gouging on the part of certain supermarket chains, whereas more profitable entities must often deal with the threat of hostile takeover by private equity firms.

A recent report by organizations representing financial institutions that have indicated that they are committed to providing financial support to sustainable agriculture, has acknowledged the existence major internal and external barriers to long-term investment in the sector (ASFI, Farming for the Future, and MACDOC, 2023).

In this kind of environment, integrated solutions that combine the provision of social housing with regenerative agriculture and circular economy initiatives, can be found by creating investment vehicles that bundle together a range of revenue streams. These up-scalable and 'fractal' developments could potentially contribute greatly to the 'crash-proofing' of those institutions offering long-term and patient investment funds. On one hand, in taking the form of unlisted equity, the investment vehicles would not suffer from upheavals in financial markets. On the other hand, the adoption of sustainable "circular economy" initiatives in networked entities could generate a stable internal market for a wide variety of goods and services, and equally stable employment opportunities, thus boosting average returns and lowering risk, while avoiding any incentives for exploitation of either income- and asset-based Ponzi schemes.

3.1 New Applications and New Initiatives

A category consists of objects and arrows that go between them. The arrows, which are also called morphisms, can be thought of as functions with a source and a target. These functions take an argument of type A then return an object of type B. They have algebraic properties including that of being associative, possess an identity operation between source and target, and are often symmetric when a given pair of functional operations are swapped around. Morphisms between arrows are called functors, and morphisms between functors are called natural transformations. The attributes of objects are defined and preserved by functors between arrows, conforming to a set of axioms, which vary according to the category under consideration.

The aim of this section of the paper will be to review developments in "applied category theory" that have contributed to the design and implementation of improved modelling capabilities based on *dynamical systems*, as well as more flexible and more user-friendly software architectures for *machine learning*.

Notable examples of this include the establishment of *Symbolica AI* by former Tesla engineer George Morgan, who has just completed a US$30m capital raising grounded in categorical deep learning principles. His researchers have deployed the **Para** construction used in lens theory, together with transfinite induction, to assemble long chains of automatons within deep learning neural networks.

In a 2021 paper on *Leaners' Languages* David Spivak (2021: 15), Chief Scientist at the MIT's Topos Institute, noted that the simple lenses used in the functional programming language, Haskell, $\mathbf{SLens}(A,B) \coloneqq \{(f_1, f^{\#}) | f_1: A \to B, f^{\#}: A \times B \to A\}$ relates inputs A to outputs with the former being updated in a manner that is sensitive to the outputs.

He contrasts this simple formulation with a map $A \to B$ within the category of parametric lenses $\mathbf{Para}(\mathbf{SLens})$ consists of a set P of parameters and functions $|f_1: A \times P \to B, f^{\#}: A \times B \times P \to A \times P$. The parameterizing set is often called "the space of weights and biases". However, in a deep learning context, the function $f^{\#}$ decomposes into the following set of bijective mappings (Fong et al, 2019: 15):

- Implement: $I: A \times P \to B$
- Update: $U: A \times B \times P \to P$
- Request: $A \times B \times P \to A$

The implement function is a P–parameterized function $A \to B$, from inputs to outputs, whereas the update and request functions take a pair (a, b) of "training data" and respectively update the parameter (by gradient descent) and return an element of the input space, which is used to train another such function in the network. Each of these functions plays an important role in the back-propagation process applied in a machine learning context (Fong et al., 2019).

Spivak (2021 19) goes on to note that a polynomial-coalgebra (S, β) can be identified with a map of polynomials $Sy^S \to p$. He then interprets this coalgebra as a simple automaton called a Moore machine, which can be given a lens representation: for sets A, B, an (A, B)-Moore machine consists of: (i) a set S, elements of which are called *states*; (ii) a readout function $r: S \to B$; and, (iii) an update function $u: S \times A \to S$, which is initialized if equipped with an element $s_0 \in S$, and can be identified with a map of polynomials $Sy^S \to By^A$ (thus, by a By^A-coalgebra $S \to By^A \lhd S$, where the symbol $\lhd$ represents composition of morphisms). Spivak observes that this mapping supports recursion, as shown in the following composite of mappings:

$$S \xrightarrow{\beta} p \lhd S \xrightarrow{p\beta} p \lhd p \lhd S \ldots$$

The Moore machine is interpreted as a controlled plant in the Figure 2, where the wiring diagram itself encloses three smaller boxes representing the controller, the plant, and the system; with each having a monomial interface. The wiring diagram itself represents a morphism, φ, shown to the left of the wiring diagram box.

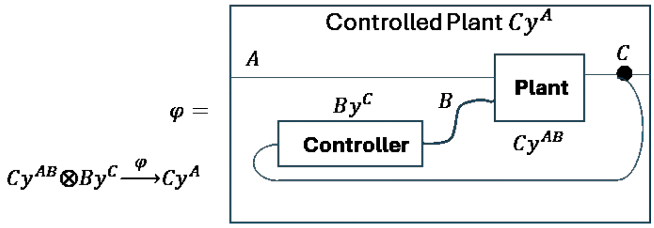


Figure 2- String Diagram for a Moore Machine, Source: Spivak (2021: 7)

If the output signal for a life just depended on the floor that someone was on, it would be a Moore Machine. If, however, the output signal also depended on the button that was pressed to indicate the floor that a person wished to travel to, then a Mealy Machine would be required. The next section of the paper will reveal that both types of automatons have been deployed as components in neural network architectures. However, another element of these new lens-based approaches will need to be described in more detail. This is the parametric lens.

The figure below depicts a parametric lens as a box using string diagrams, with 'strings' or wires flowing into and out of the box (see Cruttwell et al., 2021: 1:3). Parameter values flow in from the top while updated parameters flow out. This also applies to inputs A and outputs B.

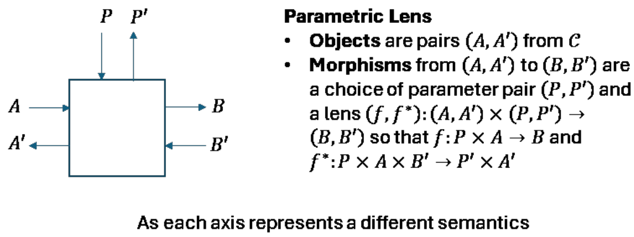


Figure 3- String Diagram Representation of a Parametric Lens

Building on seminal paper by Guillaume Boisseau (2021), Cruttwell et al., (2021: Fig. 2, 1:12), demonstrate the value of the **Para** construction, by revealing how each of the components of a deep learning neural network can be represented as links in a chain of parametric functors as depicted in Figure 4.

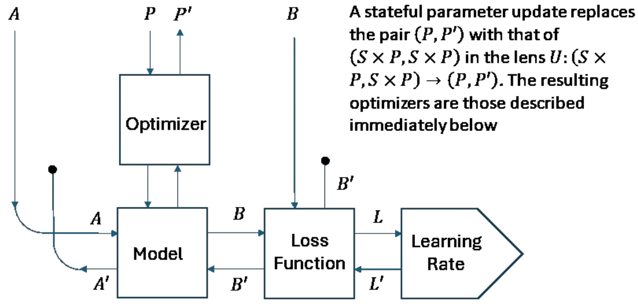


Figure 4- The Learning Process as a Sequence of Parametric Lens

Spivak has also actively promoted applications of what he calls the **Org** category (for 'organisation'), which exploits the flexibility of polynomial functors and coalgebras to formalize 'agent-environment' interactions within complex dynamical systems (Libkind and Spivak, 2024). For example, Libkind and Spivak (2024) use the **Org** category to represent the relationship between what they describe as 'patterns' and 'matter'. The pattern (represented by a free monad) unfolds in a tree-like manner. They consider four examples of patterns in accordance with this formalism: interviews, computer programs, voting schemes, and games, noting that when the monad and comonad are extended to **Org**, which is a double category, the associated Kliesli category calls multiple subprocesses before returning, while the coKliesli-category offers the ability for different machines to operate *at different rates* within wiring diagrams.

Researchers at the Topos Institute (Cappuci et al., 2024) have been using both the **Para** construction and the **Org** category to sequentially represent and solve Hamiltonian and steepest descent optimisation problems in open energy-driven physical systems (that are ubiquitous in mathematical physics). They construct a smooth space, continuous-time variant of Spivak's **Org** category for this purpose, which gives rise to a projection functor, they call a 'collapse' functor, inducing the action of a parametric lens over the system of ordinary differential equations. The action of this collapse functor is illustrated in the Figure below:

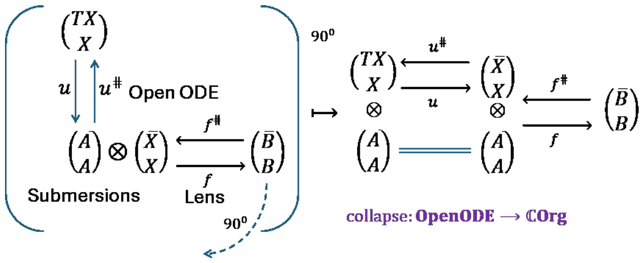


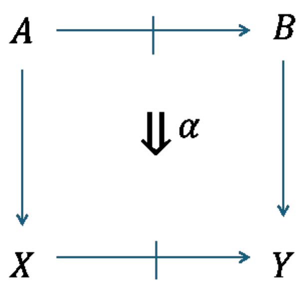
Figure 5- The Para Construction and Open Energy-Driven Systems

The fact that the system can be processed in this way, eliminates any requirement to separately account for algebraic equations that would otherwise operate as a messy set of complicated constraints over the original differential algebraic equation system under consideration. At the Field Institute's Centre for Sustainable Development, mathematical physicist, John Baez, has assembled a team of researchers to achieve improvements in formal climate change modelling (see [Mathematics for Climate Change | Azimuth (wordpress.com)](wordpress.com))

Baez has also been working with a team of applied category theorists in the MIT's *AlgebraicJulia* group with the aim of developing software for agent-based modelling, with the intention of achieving greater rigour, replicability, and transparency. Their chosen approach draws on what are called stochastic C-set rewriting systems, and also uses the **Para** construction (Para construction), but this time it is being applied for characterising behavioural differences as well as interactions between the various agents identified in the model.

C-sets are defined in relation to a small category that serves as a schema describing the types of parts in the structure of the schema and the relations between the subparts within it (for example, the schema for a graph has edges and vertices as types and two morphisms for each edge into the set of vertices that define both the source and target of the edge). The C-set is then a functor (also called a copresheaf) going from the schema to the category of sets characterizing each object of the small category in terms of the structure and morphisms making up that schema (see Lynch, 2020).

Schemas have been constructed for graphs, half graphs, reflexive graphs, symmetrical graphs, groups, spans and cospans and so on. However, the problem with C-sets is that they cannot model data structures such as labelled graphs or structured Petri nets, where every edge and vertex has an associated element from some set of labels. Accordingly, the *AlgebraicJulia* team have introduced Attribute C-sets with schemas that possess additional morphisms that label individual elements (such as each vertex and edge in a graph, or each apex and leg of a span). These can be defined over a 2-category whose objects are themselves categories, with morphisms that are functors and natural transformations. Nevertheless, in practice they are usually defined over a double category such as **Prof**, the category whose objects are categories, whose horizontal morphisms are functors, and whose vertical morphisms are profunctors. The resulting two-cells X (or squares) can be composed both vertically and horizontally.



As Evan Patterson (2024) explains, the (bi)category of sets and relations fails to have many limits and colimits. Moreover, the limits and colimits it does possess behave counterintuitively due to the self-duality: products in the bicategory category (which are also coproducts) are disjoint unions of sets. Finally, the cartesian (set-theoretic) product of relations does not arise from any standard universal property, which is problematic. Double categories overcome these problems. Vertical morphisms play the role of functions but horizontal morphisms are relations. In addition, universal properties are derived from the composition of functions, of relations, and of the cells between them.

*AlgebraicJu*lia also recognizes a variety of different kinds of *operads* that also characterise the ways that mathematical structures are preserved and modified under functorial morphisms, especially those which translate directed wiring diagrams into undirected wiring diagrams, and circular port diagrams (see Libkind, 2020). The Grothendieck construction defined over directed wiring diagrams is deployed to identify dependencies and cycles. The latter can then be removed (effectively by squashing multiple dependencies into a single dependency using what is called a pullback) to form acyclic wiring diagrams. The (functorial) composition of smaller operads to form a larger one is depicted in Figure 6 (composition is subject to associativity, unitality, and symmetry axioms, with symmetry defined in relation to the pair-wise permutations of the relevant operations).

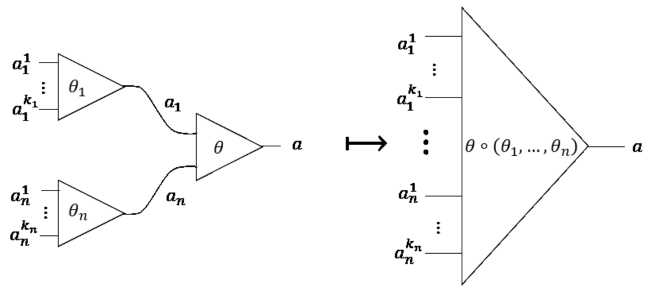


Figure 6- The (functorial) composition of smaller operads

The mathematical components described above have allowed functional programmers to accommodate a variety of different formalisms for dynamical systems within a unified and diagrammatically-based whole. This includes Jay Forester's well known stock-flow modelling approach (Baez, Li, Libkind, Osgood, and Patterson, 2022), (coloured) Petri nets with rates that have been applied to industrial and computational processes as well as epidemiological modelling, as well as the agent-based modelling approach made famous by researchers at the Sante-Fe institute (Brown, 2023; Baez, 2024), along with Tonti diagrams (Patterson et al., 2022).

Ther Quantinuum Group, currently engaged in a US$300m capital raising, is the outcome of a merger between the Cambridge Quantum Computing corporation and a division of Honeywell. This organization, which boasts five research centres across the UK and Europe, specializes in applications of the ZX and ZW diagrammatic calculi to both quantum chemistry and quantum computing systems (see de Wetering, 2020).

3.2 Implications for Dynamical system modelling

David Myers (2022) has introduced categorical 'doctrines' as a way to formally characterize the compositional features that are associated with diagrammatic representations of dynamical systems. These include signal flow diagrams, wiring diagrams, and bubble diagrams (also called circular port diagrams), each characterized by a unique doctrine. By applying certain functors, analysts can then move from one system doctrine to another as required. For example, signal flow graphs can be constructed (Figure 7) from the following five morphisms defined over an object k (Baez and Erbele, 2014).

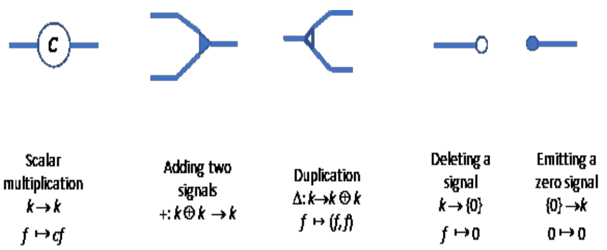


Figure 7- Signal Flow Graph

Because the application of Laplace transforms can also be viewed as a scalar product operation, differential equations can be represented by these five operators. The addition of 'cup' and cap' operations that allow wires to be 'turned' around or inverted means that feedback control processes can also be expressed in diagrammatic form, as shown below in Figure 8.

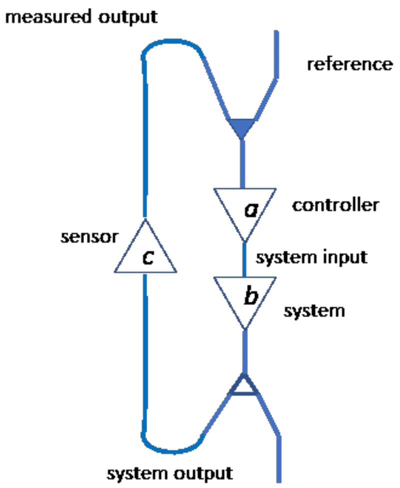


Figure 8- Feedback Control Process

The category applying to signal flow graphs is called a symmetric monoidal category because its functors preserve the arity and algebraic structure of monoids whose operations include multiplication and addition.

Petri nets can also be represented diagrammatically because they consist of three types of nodes (input nodes, output nodes and transitions), which can be connected by wires. Transitions will only 'fire' when the requisite number of up-stream inputs has been made available. On completion of the transition, products are then conveyed on wires to any attached down-stream nodes. Inputs (and outputs) are recorded. The set of available inputs at any one time is called a 'mark'. They can also be 'coloured' or distinguished from one another by pertinent labels. Finally, when requisite inputs have either been produced or otherwise made available, the rate at which any transition fire can be defined by parameters drawn from the well-known exponential or 'queuing' distribution.

When applied to reaction-diffusion systems, under the law of mass action inputs are transformed into outputs at rates proportional to the product of input concentrations. In this case, Petri nets are viewed as a composite of sub models, with the syntax for composition given by undirected wiring diagrams.

The doctrinal approach favoured by David Myers (2022) also makes computational and diagrammatic use of the optics, lenses and charts familiar to software engineers. When these devices are applied to dynamical systems they, respectively, assist in the interpretation of bijective flows such as gradient descent or Hamiltonian optimization, or the dynamic characteristics of any resulting cycles and orbits.

An alternative approach to using a category like this would be to deploy symbolic algebra to represent functions between topological spaces, including for the specific case of open dynamical systems (Lynch, 2023).

3.3 Implications for AI and machine learning

Researchers from Google DeepMind, Symbolica AI, and the Universities of Cambridge and Edingburgh have come together to develop a general-purpose framework for specifying and constructing deep learning architectures. To this end, they extend the existing categorical approach to machine learning to capture a wider range of different data structures and components viewed as links in a chain of parameterized automatons. In particular, they have embraced more structured monads and their algebras, which are typically embedded in coalgebras of endofunctors, by working in the 2-category of parametric functors, which are 1-morphisms, and 2-morphisms that capture reparametrisations between parametric functors. The associated categorical algebra homomorphisms are deployed to generalize both the *top-down* equivariance constraints and permutation-based equivariant learning over directed graphs (associated with software such as **PyG**) and *bottom-up* processes of automatic differentiation (associated with software such as **PyTorch** and **TensorFlow).** based on. The resulting monad algebras thus generalise equivariance constraints, enabling them to be both parametric and lax.

Kelly's transfinite construction is then deployed to derive folding and unfolding recursive neural networks, as well as networks of Mealy and Moore machines by iteration as shown in the Figure 9.

Jost one example of the resulting chains, populated with Moore machines, is shown in Figure 10.

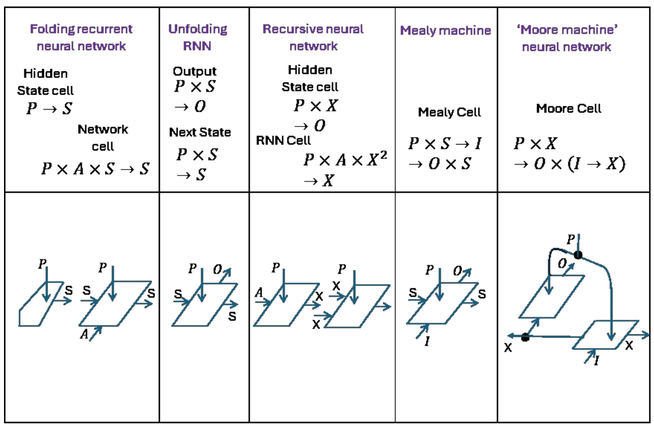


Figure 9- *Architectures for Categorical Deep Learning via the Para Construction*, Source: Gavranović et al., 2024.

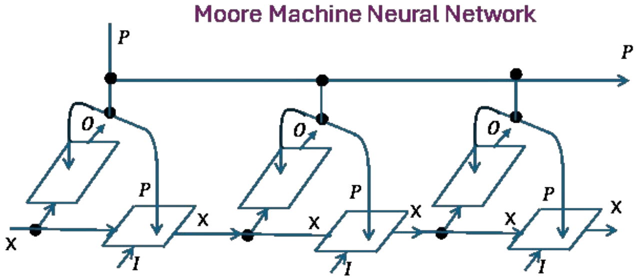


Figure 10- Moore Machine Neural Network

The authors observe that the resulting networks will not only support reparameterization, but also non-linear maps as well as weight-sharing, along with verifiably correct logical arguments or code These networks will also possess an internal logic and the dependent type theory associated with a topos, as the associated mathematical site. These will allow the types of output to become a function of the value of inputs (allowing for logical proofs and not just numbers). In his PhD thesis, with potential to improve regulation.

3.4 New Extensions to Graphical Calculi

Collaborative and cross-disciplinary forms of model construction, calibration and simulation can be supported and enhanced through the active deployment of diagrammatic forms of model design and reasoning. As applied category theory has evolved, string diagrams have evolved as well. Petri nets have always relied for their construction on user-friendly diagrammatic representations and as we have seen, this also holds for neural network architectures. Engineers are used to reasoning with signal flow graphs and additional graphical variants have been added to this familiar construct. Algebraic Julia has been designed to allow coding to be replaced by the use of string diagrams, including the stock-flow diagrams developed by Jay Forrester.

String diagrams themselves have been extended in two main ways: (1) they have been transformed from two-dimensional strings into three dimensional surfaces; and, (2) they have incorporated a variety of labelled functor 'boxes' to characterise more complex forms of functorial morphisms, indexed sets (including those that might be infinite), and equivalence structures. In the interests of brevity, I will now confine my discussion to three examples of the latter.

Nick Hu has drawn on a technique developed for topological quantum field theory that combines both *internal and external diagrams* (see Hu and Vicary, 2021). The internal sting diagrams, which go from the top down, represent a monoidal category with tensor product. The right-adjoint for this product morphism is a freely generated presentation. The external diagrams take the form of hollow tubes, whose morphism go from the bottom up, and conform to a profunctor calculus, which can be decomposed into a set of 1-morphism generators. Hu extended this diagrammatic machinery to traced monoidal categories, so that he could account for recursion and processes of feedback control. Without getting into technicalities, Figure 11 depicts what this traced monoidal process looks like diagrammatically:

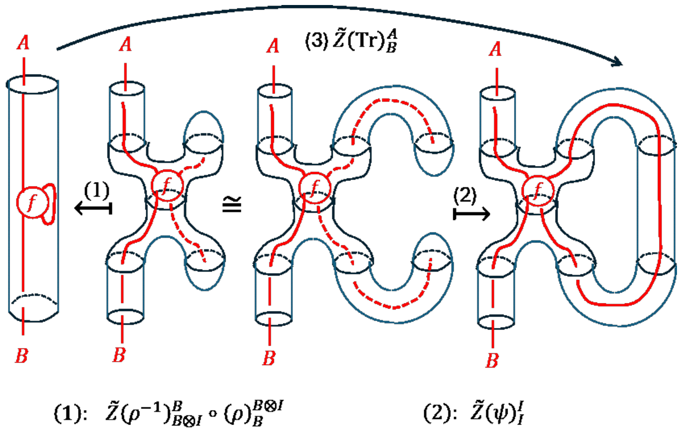


Figure 11- External Traced Monoidal Categories [Hu, 2019:30]

Vince Abbott's Honors thesis has the aim of developing *robust diagrams* for deep learning neural networks, which encompass convolution neural networks, transformer models, residual networks, and other new modalities (see Abbott and Zardini, 2021). To this end, he has taken advantage of extensions to the string diagrammatic calculus that have already been made by Kenji Nakahira (2023). Two principles underpin these functor string diagrams: (i) vertical section decomposition of diagrams to account for morphisms imposed by, or applied between pairs of functors; and (ii) equivalent expressions depicted to provide graphical intuition. Colours are deployed in the diagrams for annotation rather than expression, and new features are introduced where necessary "to reveal the algebraic structure in a range of cases, from formal proofs to the design of deep learning model" (Abbott and Zardini, 2021: 2).

Ghica and Zanasi (2023: 4) adopt a 'syntactic trinitarian' approach based on terms, diagrams, and graphs, with each being effectively interchangeable representations of an abstract "ideal of syntax, each serving distinct roles." The aim of their diagrammatic approach is to examine Cartesian Closed Categories as a formal basis for the lambda calculus that can subsequently be refined into (inductive) graph-algorithms. Accordingly, it is situated mid-way between sequential terms and graphs and stems from both planar and string diagrammatic syntax.

4. Conclusions

Optics and lenses have a wide variety of applications to dynamical systems and machine learning. In each case they possess a valuable diagrammatic representation (as string diagrams), which can operate as an aid in their software-based development and deployment (e.g. AlgebraicJulia, Symbolica AI, Haskell applications).

Theoretical and practical developments that are currently underway should assist users in the collaborative modelling of the *Polycrisis* by: (i) integrating techniques of, and new architectures for, machine learning, differential programming, and calibration and simulation of dynamical systems; (ii) accounting for subsystem interactions that may each possess different stock-flow rates within a larger system; (iii) accommodating non-linear mappings and weight-sharing in machine learning; (iv) encompassing both Forrester's stock-flow- and the Sante Fe Institute's agent-based-modelling of dynamic systems; (v) formally supporting a variety of stochastic influences over the systems that are being modelled, while imposing on the whole, an internal logic.

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